

Cambridge  
International  
AS & A Level

**Cambridge International Examinations**  
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE  
NAME

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CENTRE  
NUMBER

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**MATHEMATICS**

**9709/12**

Paper 1 Pure Mathematics 1 (P1)

**October/November 2018**

**1 hour 45 minutes**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **20** printed pages.



1 Find the coefficient of  $\frac{1}{x^2}$  in the expansion of  $\left(3x + \frac{2}{3x^2}\right)^7$ . [4]

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2 Showing all necessary working, find  $\int_1^4 \left(\sqrt{x} + \frac{2}{\sqrt{x}}\right) dx$ . [4]

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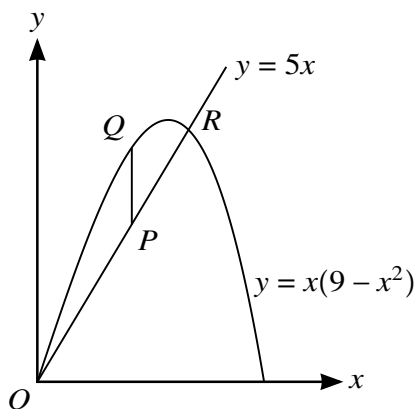
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The diagram shows part of the curve  $y = x(9 - x^2)$  and the line  $y = 5x$ , intersecting at the origin  $O$  and the point  $R$ . Point  $P$  lies on the line  $y = 5x$  between  $O$  and  $R$  and the  $x$ -coordinate of  $P$  is  $t$ . Point  $Q$  lies on the curve and  $PQ$  is parallel to the  $y$ -axis.

- (i) Express the length of  $PQ$  in terms of  $t$ , simplifying your answer. [2]

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- (ii) Given that  $t$  can vary, find the maximum value of the length of  $PQ$ . [3]

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4 Functions  $f$  and  $g$  are defined by

$$f : x \mapsto 2 - 3 \cos x \quad \text{for } 0 \leq x \leq 2\pi,$$

$$g : x \mapsto \frac{1}{2}x \quad \text{for } 0 \leq x \leq 2\pi.$$

(i) Solve the equation  $fg(x) = 1$ . [3]

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(ii) Sketch the graph of  $y = f(x)$ . [3]



**(ii)** Find the fourth term of each progression.

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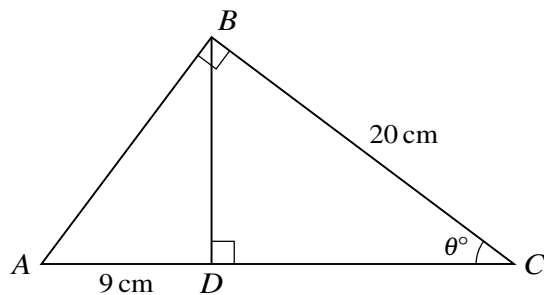
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The diagram shows a triangle  $ABC$  in which  $BC = 20$  cm and angle  $ABC = 90^\circ$ . The perpendicular from  $B$  to  $AC$  meets  $AC$  at  $D$  and  $AD = 9$  cm. Angle  $BCA = \theta^\circ$ .

- (i) By expressing the length of  $BD$  in terms of  $\theta$  in each of the triangles  $ABD$  and  $DBC$ , show that  $20 \sin^2 \theta = 9 \cos \theta$ . [4]

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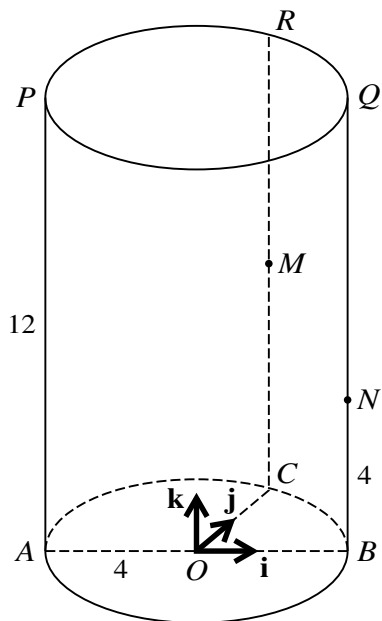
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The diagram shows a solid cylinder standing on a horizontal circular base with centre  $O$  and radius 4 units. Points  $A$ ,  $B$  and  $C$  lie on the circumference of the base such that  $AB$  is a diameter and angle  $BOC = 90^\circ$ . Points  $P$ ,  $Q$  and  $R$  lie on the upper surface of the cylinder vertically above  $A$ ,  $B$  and  $C$  respectively. The height of the cylinder is 12 units. The mid-point of  $CR$  is  $M$  and  $N$  lies on  $BQ$  with  $BN = 4$  units.

Unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are parallel to  $OB$  and  $OC$  respectively and the unit vector  $\mathbf{k}$  is vertically upwards.

Evaluate  $\overrightarrow{PN} \cdot \overrightarrow{PM}$  and hence find angle  $MPN$ . [7]

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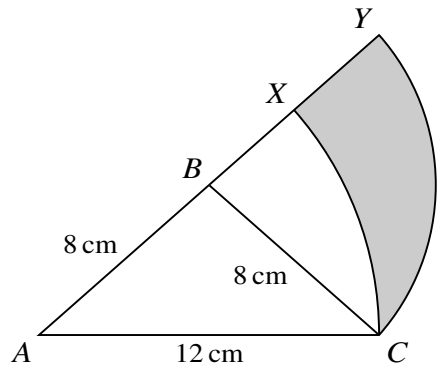
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The diagram shows an isosceles triangle  $ACB$  in which  $AB = BC = 8\text{ cm}$  and  $AC = 12\text{ cm}$ . The arc  $XC$  is part of a circle with centre  $A$  and radius  $12\text{ cm}$ , and the arc  $YC$  is part of a circle with centre  $B$  and radius  $8\text{ cm}$ . The points  $A, B, X$  and  $Y$  lie on a straight line.

- (i) Show that angle  $CBY = 1.445$  radians, correct to 4 significant figures. [3]

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(ii) Find the perimeter of the shaded region.

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9 The function  $f$  is defined by  $f : x \mapsto 2x^2 - 12x + 7$  for  $x \in \mathbb{R}$ .

(i) Express  $2x^2 - 12x + 7$  in the form  $2(x + a)^2 + b$ , where  $a$  and  $b$  are constants. [2]

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(ii) State the range of  $f$ . [1]

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The function  $g$  is defined by  $g : x \mapsto 2x^2 - 12x + 7$  for  $x \leq k$ .

**(iii)** State the largest value of  $k$  for which  $g$  has an inverse. [1]

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**(iv)** Given that  $g$  has an inverse, find an expression for  $g^{-1}(x)$ . [3]

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10 The equation of a curve is  $y = 2x + \frac{12}{x}$  and the equation of a line is  $y + x = k$ , where  $k$  is a constant.

(i) Find the set of values of  $k$  for which the line does not meet the curve. [3]

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In the case where  $k = 15$ , the curve intersects the line at points  $A$  and  $B$ .

(ii) Find the coordinates of  $A$  and  $B$ . [3]

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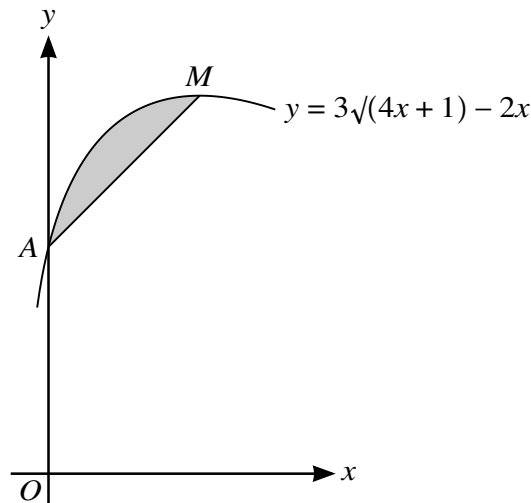


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**(iii)** Find the equation of the perpendicular bisector of the line joining  $A$  and  $B$ . [3]

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The diagram shows part of the curve  $y = 3\sqrt{4x + 1} - 2x$ . The curve crosses the y-axis at  $A$  and the stationary point on the curve is  $M$ .

- (i) Obtain expressions for  $\frac{dy}{dx}$  and  $\int y dx$ . [5]

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(ii) Find the coordinates of  $M$ .

[3]

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(iii) Find, showing all necessary working, the area of the shaded region.

[4]

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